

Reply to the comment on 'Boson-fermion model beyond the mean-field approximation'

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## REPLY TO COMMENT

## Reply to the comment on ‘Boson–fermion model beyond the mean-field approximation’

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**Abstract.** There is no Cooper pairing in the boson–fermion model (BFM) if bosons are not condensed. There is no Bose–Einstein condensation and, consequently, no Cooper pairing in two-dimensional BFM. The Cooper pairing is possible in a three-dimensional BFM if and only if bosons are condensed but can hardly be realized because of a charge-order instability.

The boson–fermion model (BFM) with initially localized (dispersionless) bosons hybridized with free fermions was motivated by the alleged difficulty of accommodating a stable *mobile* bosonic field. It was originally studied in the framework of the mean-field approximation (see, for example, references [2] and [3] of the preceding comment [1]) including a two-dimensional case [2]. One result was that ‘a BCS-like superconducting state in the fermionic subsystem occurred via fermion pairs being virtually excited into the unoccupied bosonic states’ (reference [3] of the comment), in which case the superconducting transition was believed to be almost conventional [2] with a very high  $T_c$  of the order of the Fermi temperature. However, our study of the BFM beyond the mean-field approximation [3, 4] shows that:

- (a) there is no Cooper pairing of fermions without a condensation of real bosons [3];
- (b) there is no condensation of bosons and therefore no pairing of fermions at any temperature in two dimensions (fermions in planes), i.e.  $T_c = 0$  [4];
- (c) the critical temperature of the Bose–Einstein condensation of the 3D BFM is very low, less than 1 K, and the inverse lifetime of the long-wave bosons is of the order of their energy [3, 4];
- (d) the Cooper pairing is possible if and *only if* bosons are condensed [4].

We proved the last statement by applying the most divergent ‘ladder’ approximation for the boson self-energy and for the fermion vertex.

In their comment [1] Friedberg *et al* point out an exact identity relating the boson propagator to the fermion  $t$  matrix, confirming that our statement (d) is exact. Although the comment does not invalidate any of our results, (a)–(d), by strengthening (d) it supports the opinion that the BFM with the initially localized bosons is a superfluid. We proved that this is not the case in 2D [4] and we do not believe that a superfluid state of BFM may be realised in three dimensions. Firstly, the boson self-energy has a sizable imaginary part which is linear in energy [3]. This suggests that the low-energy bosonic modes of BFM are damped and cannot propagate (i.e. no superfluidity). Secondly, before going beyond the most divergent (‘ladder’) approximation in the Cooper channel one should consider an instability of other channels. The ‘bandwidth’ of localized bosons  $z_c$  due to their hybridization with fermions

was estimated by us as  $z_c \sim v^2 N(0)$  which a few kelvin or less [3]. Therefore, one can expect a commensurate or incommensurate charge ordered (CO) state of BFM, rather than a superfluid for any sizable boson density  $n_b \geq 0.1$  (per site) [4]. One can estimate the corresponding critical temperature  $T^*$  of the CO instability by the use of the  $T$ - $n$  phase diagram of the hard-core bosons on a lattice (or of the Heisenberg pseudospin Hamiltonian) as

$$T^* \simeq 4V_c n_b \quad (1)$$

where  $4V_c$  is the Coulomb repulsion of bosons. Definitely,  $T^*$  is above the Bose–Einstein condensation temperature  $T_c \leq z_c$  by several orders of magnitude.

There is also a fundamental problem with any theory involving real-space pairs (bosons) tightly bound by a field of a pure electronic origin. As stressed by Emery *et al* [5] such theories are *a priori* implausible due to the strong short-range Coulomb repulsion between two carriers. A direct (density–density) repulsion is usually much stronger than any exchange interaction. The attraction potential generated by the electron–phonon interaction of the Holstein model may overcome the short-range Coulomb repulsion, but inevitably involves a huge carrier mass enhancement, otherwise the phonon frequency would be extremely high. On the other hand, we have shown [6] that the Fröhlich electron–phonon interaction can provide *intrinsically* mobile intersite small bipolarons, which are condensed at high  $T_c$  of the order of 100 K. We believe that *only* this interaction operating on a scale of the order of 1 eV can compensate the intersite Coulomb repulsion allowing the deformation potential (together with an exchange interaction of any origin) to bind two holes into an intersite mobile bipolaron in the  $\text{CuO}_2$  plane. The bipolaron mass renormalization appears to be smaller by several orders of magnitude than in the Holstein model with the same value of the attraction potential. Our conclusion on the nature of high- $T_c$  superconductivity is supported by a very large value of the static dielectric constant in cuprates as well as by the fact that the charge  $2e$  Bose liquid of small bipolarons describes well many of the anomalous properties of these materials both in the normal and superconducting states [7]. Although we do not exclude a coexistence of Fermi and Bose carriers in some systems (in fact, we discussed their mixture back in 1986 [8]), we have presented reasoned arguments ruling out any role which their hybridization could play.

## References

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